جامعة الجزيرة الخاصة
ALJAZEERA PRIVATE UNIVERSITY

Signal processing
معالجة الاشارة

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System Modeling

To do engineering design, we must be able to accurately predict the quantitative behavior of a circuit or other system.

This requires math models:

Circuits

Device Rules
- R: $v(t)=Ri(t)$
- L: $v(t)=L(di(t)/di)$
- C: $dv(t)/dt=1/Ci(t)$

Circuit Rules
- -KVL
- -KCL
- -Voltage Divider
- -etc.

→ **Differential Equation**

Mechanical

Device Rules
- Mass: $M(d^2p(t)/dt^2)$
- Spring: $k_x p(t)$
- Damping: $k_d(dp(t)/dt)$

System Rules
- -Sum of forces
- -etc.

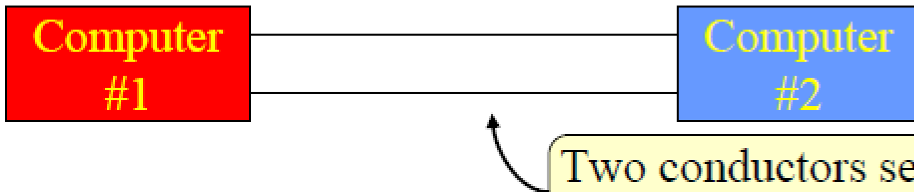
→ **Differential Equation**

Similar ideas hold for hydraulic, chemical, etc. systems…

→ **"differential equations rule the world"**

Simple Circuit Example:

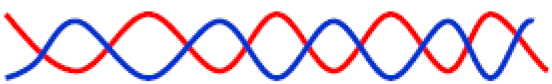
Sending info over a wire cable between two computers



Two conductors separated by an insulator
$\Rightarrow$ capacitance

Two practical examples of the cable

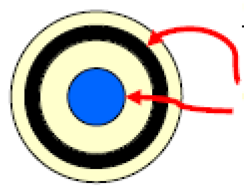
"Twisted Pair" of Insulated Wires



Typical values: 100 Ω/km

50 nF/km

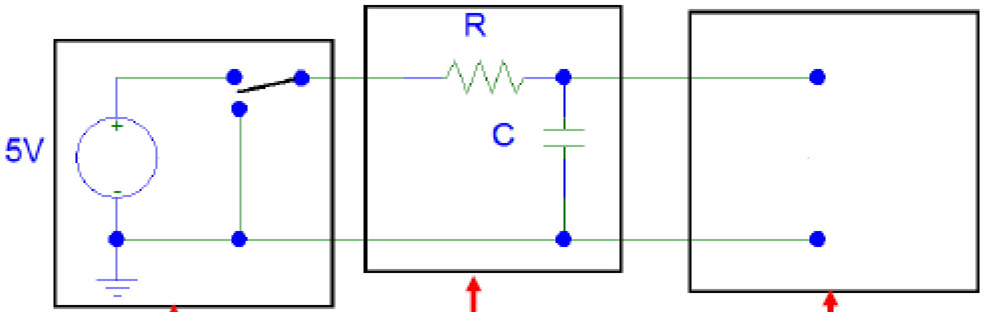
coaxial cable



conductors separated by insulator

Recall: resistance increases with wire length

Simple Model:



Driver's Thevenin Equivalent Circuit (Computer #1)

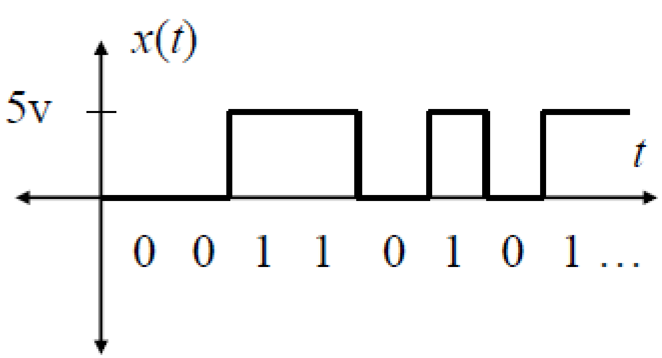
Cable Model

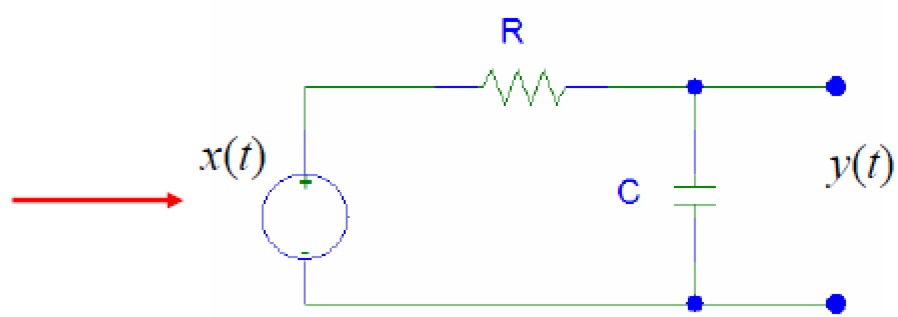
Receiver's Thevenin Equivalent Circuit (Computer #2)

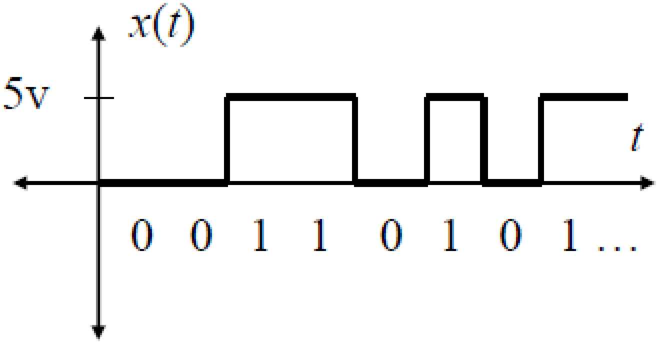
Infinite Input Resistance (Ideal)

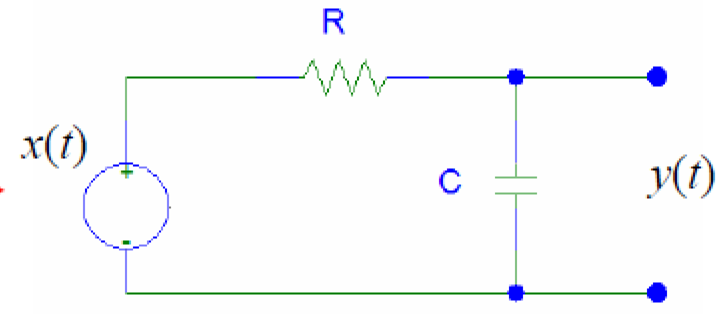
Zero Output Resistance (Ideal)

Effective Operation:









Use Loop Equation & Device Rules:

$$x(t) = v_R(t) + y(t)$$

$$v_R(t) = Ri(t)$$

$$i(t) = C\frac{dy(t)}{dt}$$

$$\frac{dy(t)}{dt} + \frac{1}{RC}y(t) = \frac{1}{RC}x(t)$$

This is the Differential Equation to be "Solved":

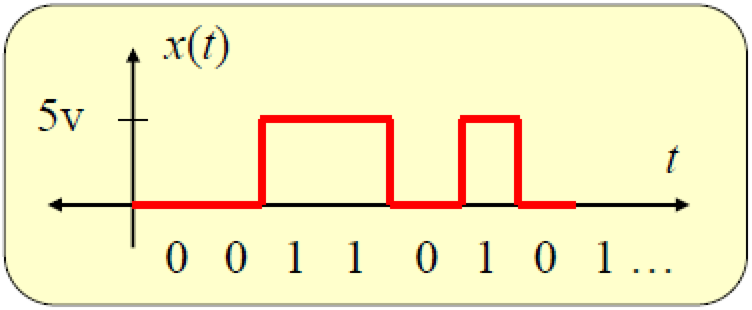
Given: Input $x(t)$ Find: Solution $y(t)$

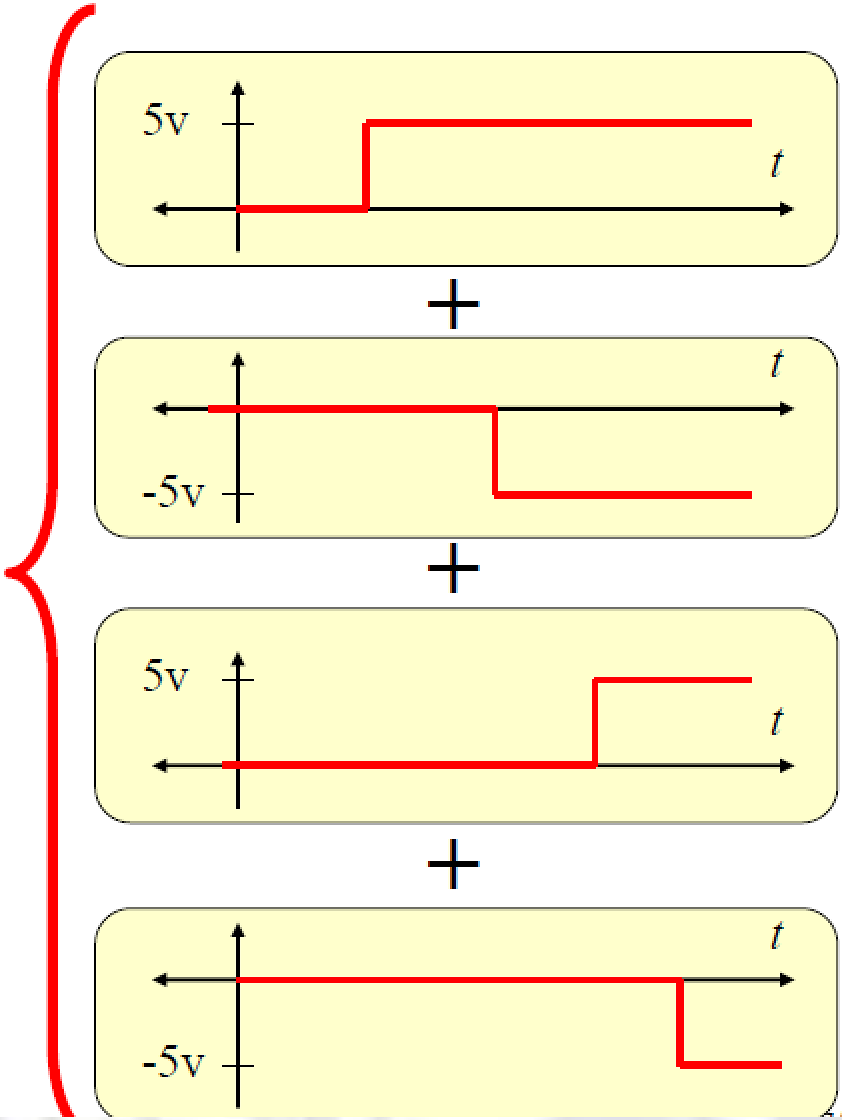
Recall: A "Solution" of the D.E. means… The function that when put into the left side causes it to reduce to the right side

Differential Equation & System … the solution is the output

Now… because this is a **linear** system (it only has *R*, *L*, *C* components!) we can analyze it by **superposition**.

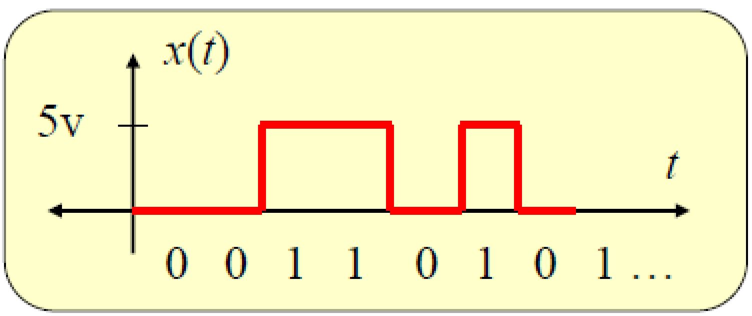
Decompose the input…

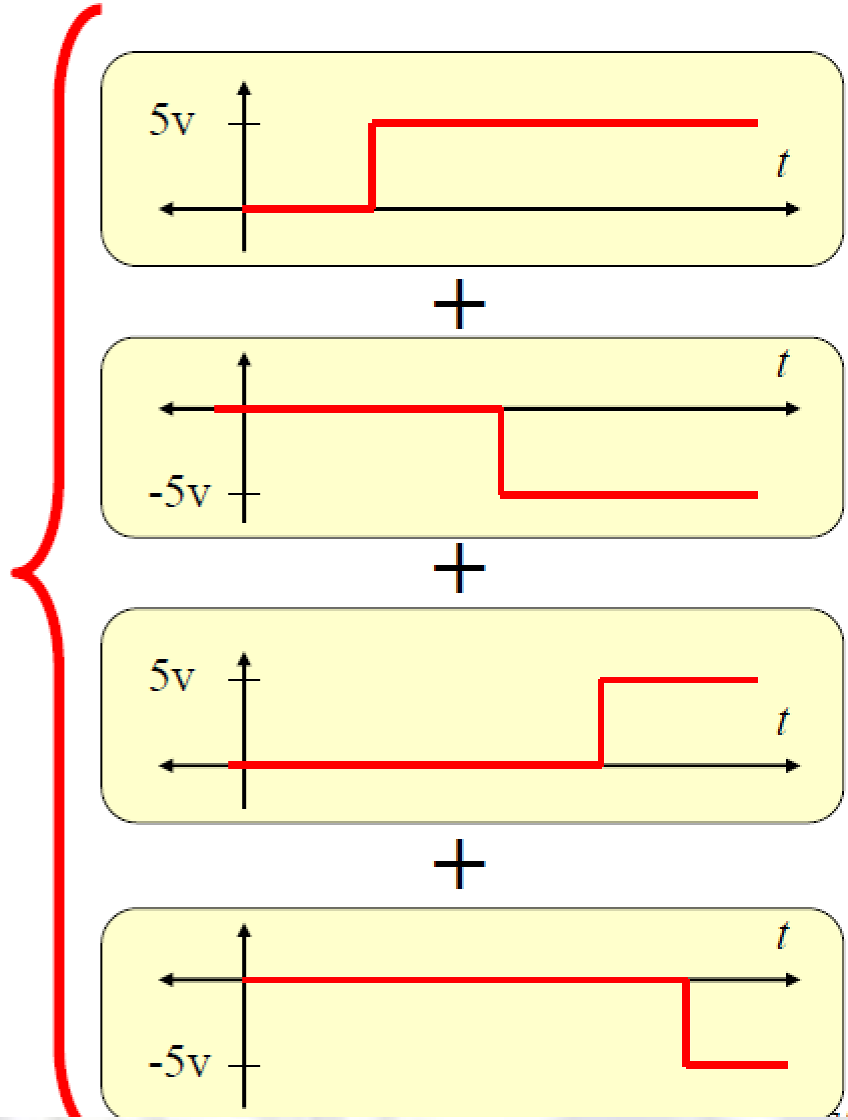




Now… because this is a **linear** system (it only has R, L, C components!) we can analyze it by **superposition**.

Decompose the input…

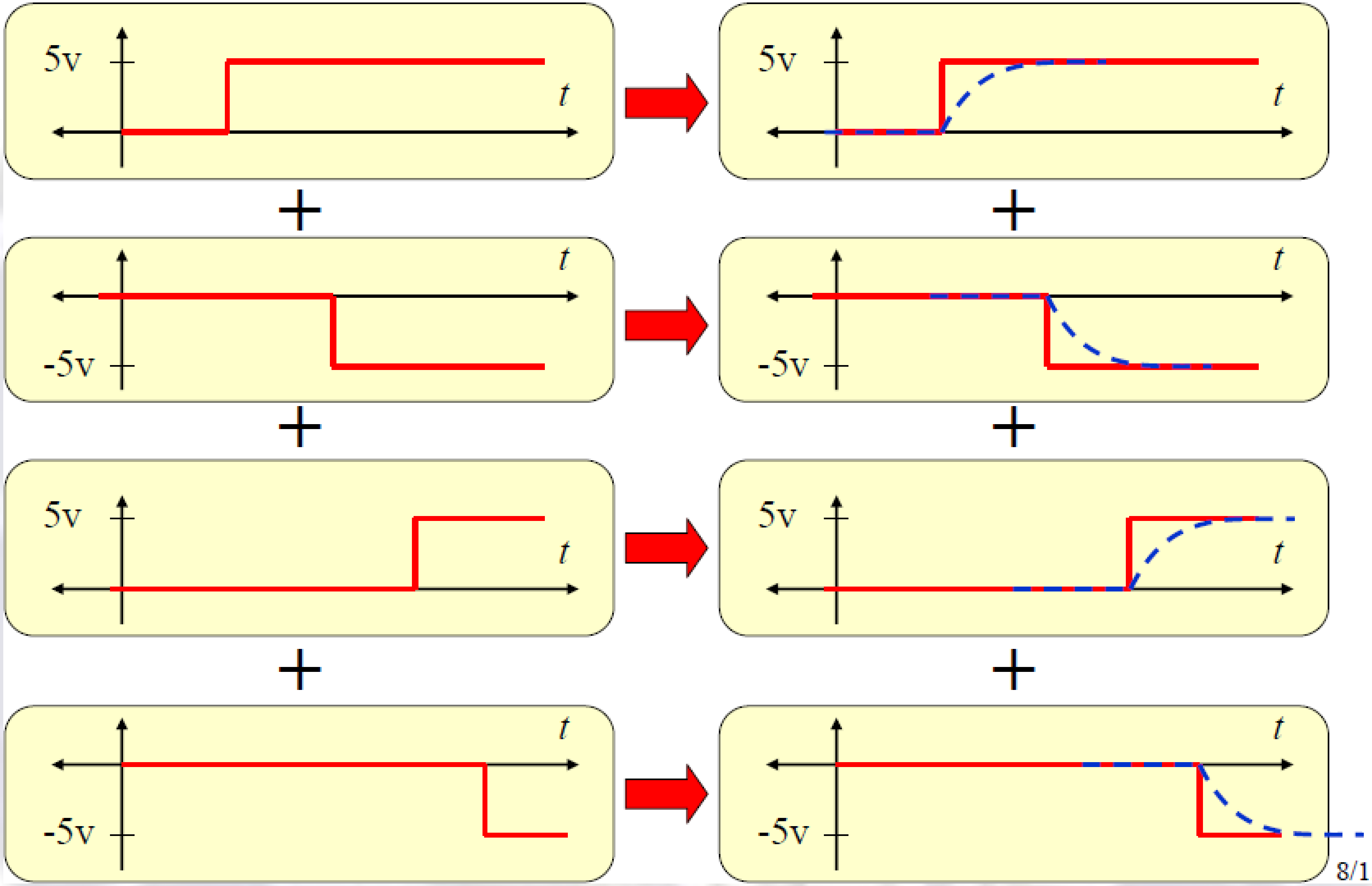




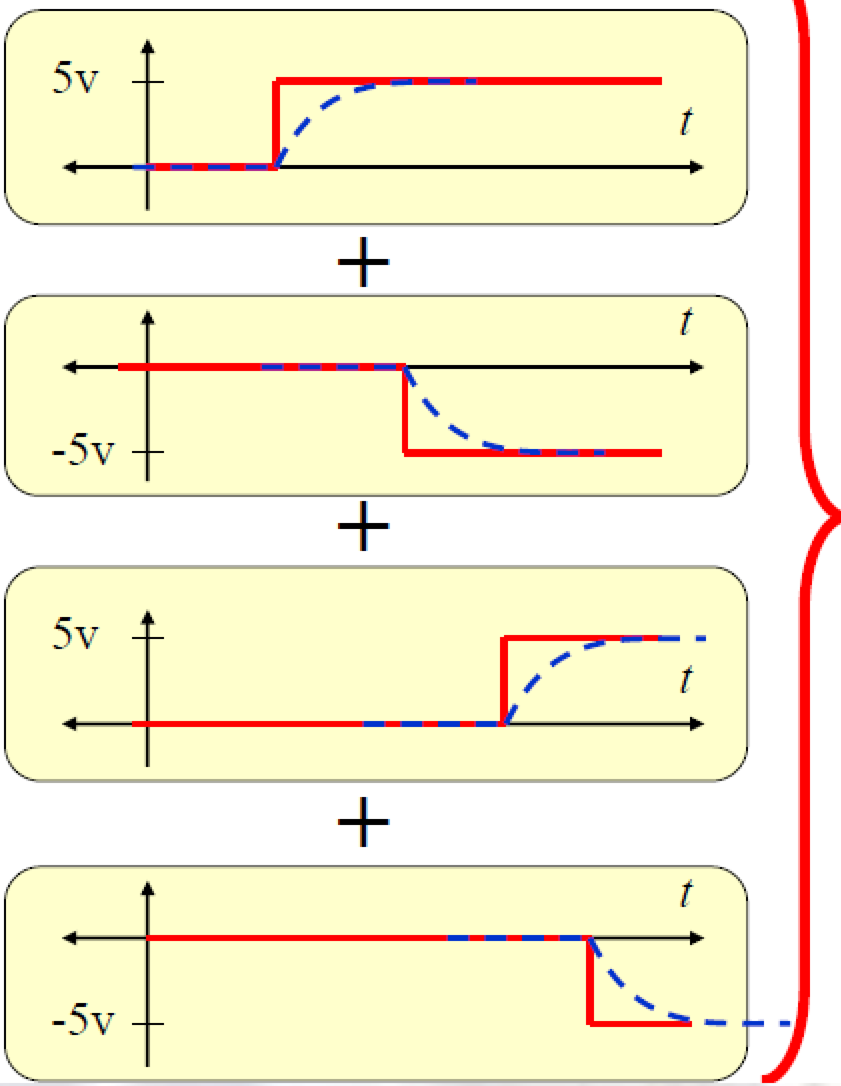
Input Components

Output Components (Blue)

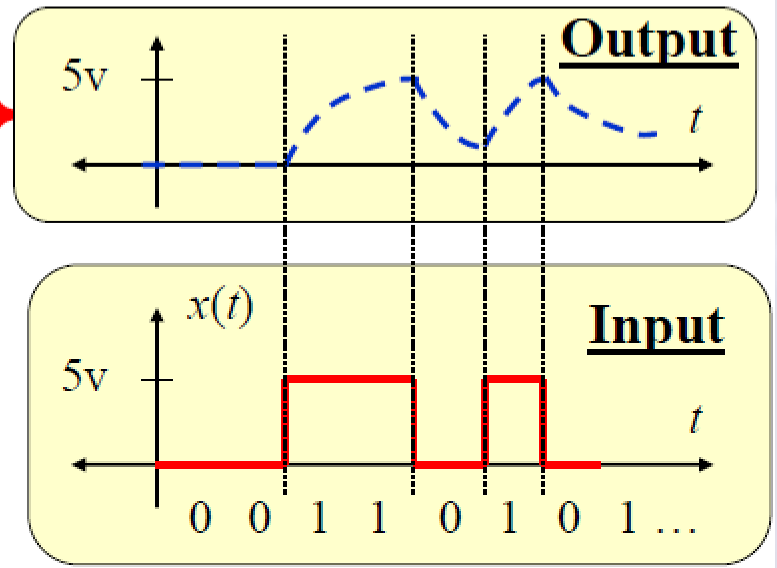
Standard Exponential Response
Learned in "Circuits":



Output Components

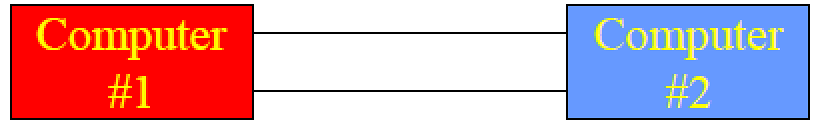


Output is a "smoothed" version of the input… it is harder to distinguish "ones" and "zeros"… it will be even harder if there is noise added onto the signal!

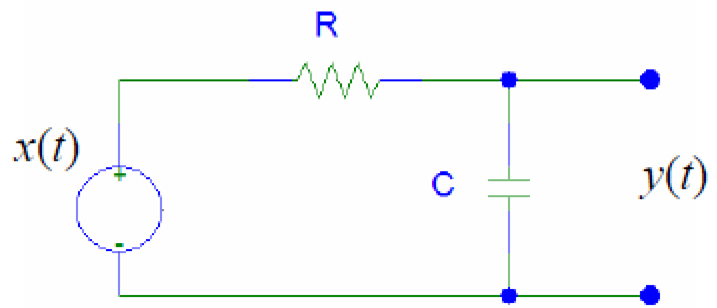


Progression of Ideas an Engineer Might Use for this Problem

Physical System:



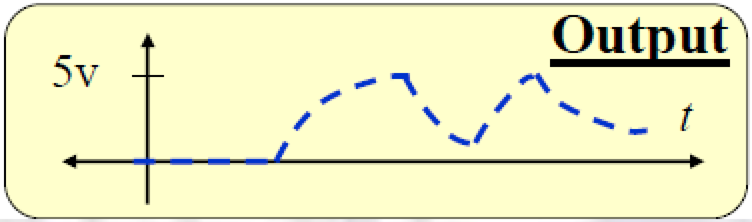
Schematic System:



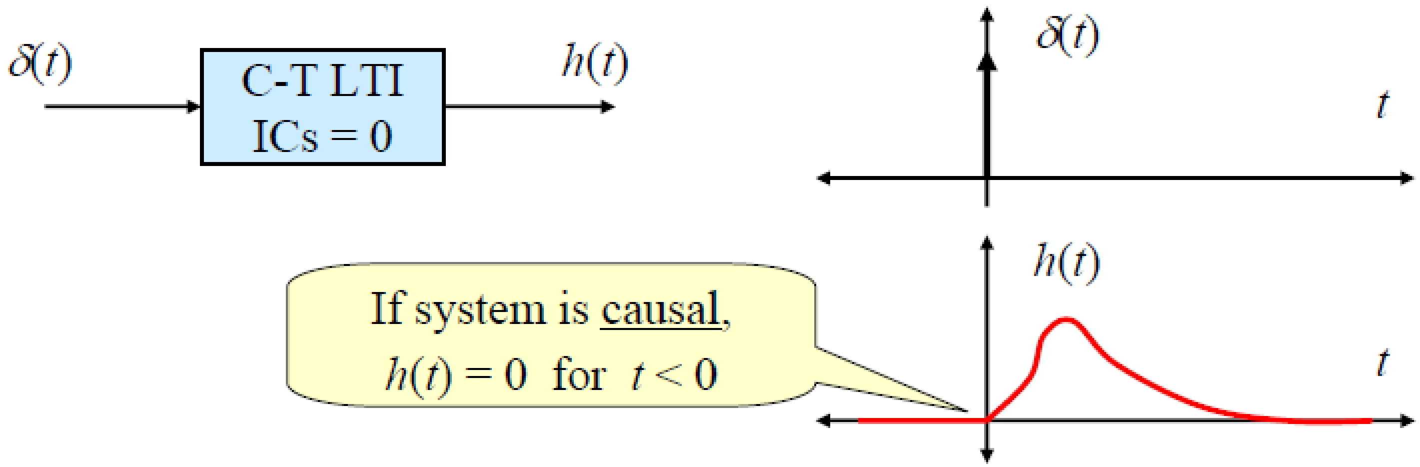
Mathematical System:

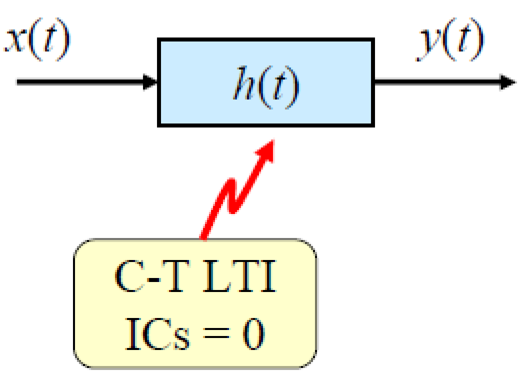
$$\frac{dy(t)}{dt} + \frac{1}{RC}\,y(t) = \frac{1}{RC}\,x(t)$$

Mathematical Solution:



Impulse Response: $h(t)$ is what "comes out" when $\delta(t)$ "goes in"



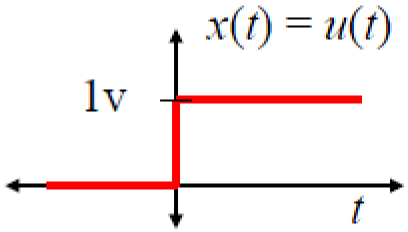


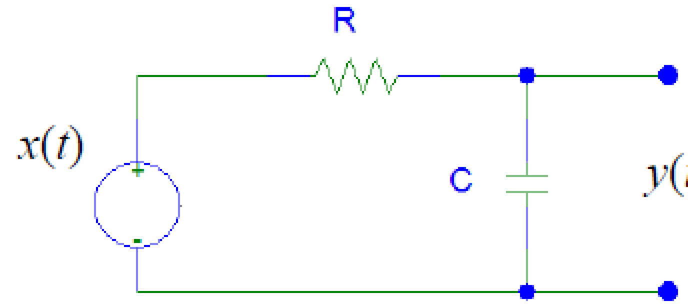
CONVOLUTION

$$y(t) = \int_{-\infty}^{\infty} x(\lambda)h(t-\lambda)d\lambda$$

Notation: $y(t)=x(t)*h(t)$

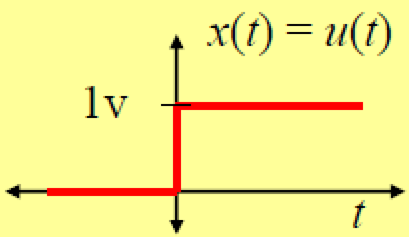
For our step input:





$y(t) = x(t) * h(t)$

This is the convolution we need to do…



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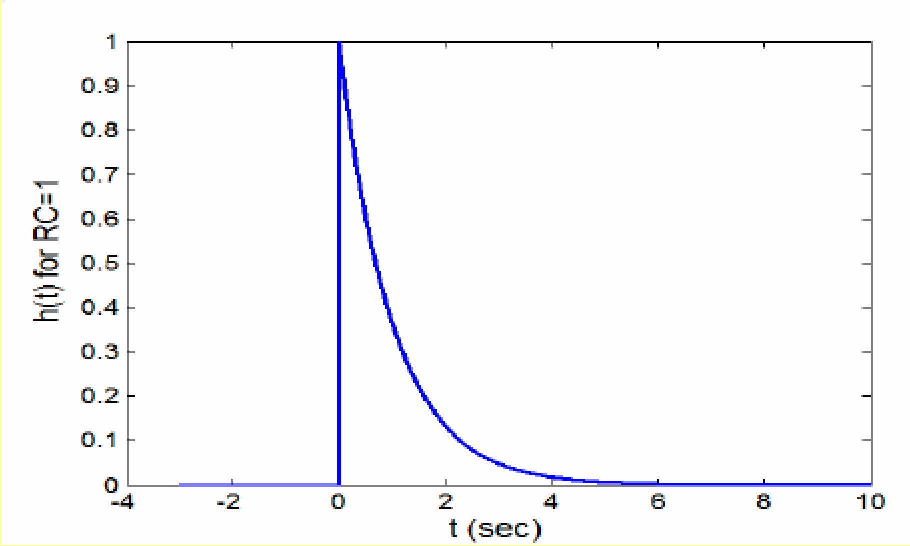
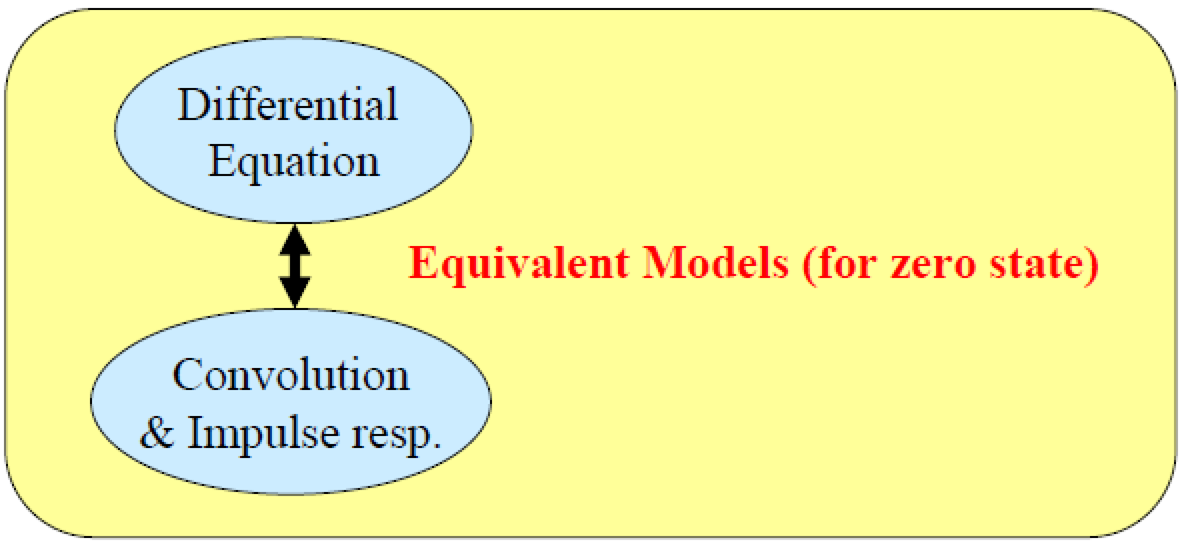


For a LTI C-T system in <u>zero state</u> we no longer need the differential equation model…

-Instead we need the impulse response $h(t)$ & convolution

New alternative model!



C-T convolution properties

Many of these are the same as for DT convolution.

We only discuss the new ones here.

See the next slide for the others

1. **Derivative Property:**

derivative

$$\frac{d}{dt}[x(t)*v(t)] = \dot{x}(t)*v(t)$$

$$= x(t)*\dot{v}(t)$$

2. **Integration Property** Let $y(t) = x(t)*h(t)$, then

$$\int_{-\infty}^{t} y(\lambda)d\lambda = \left[\int_{-\infty}^{t} x(\lambda)d\lambda\right]*h(t) = x(t)*\left[\int_{-\infty}^{t} h(\lambda)d\lambda\right]$$

Convolution Properties

These are things you can exploit to make it easier to solve convolution problems

1. Commutativity $x(t) * h(t) = h(t) * x(t)$

$\Rightarrow$ You can choose which signal to "flip"

2. Associativity $x(t) * (v(t) * w(t)) = (x(t) * v(t)) * w(t)$

$\Rightarrow$ Can change order $\rightarrow$ sometimes one order is easier than another

3. Distributivity $x(t) * (h_1(t) + h_2(t)) = x(t) * h_1(t) + x(t) * h_2(t)$

$\Rightarrow$ may be easier to split complicated system $h[n]$ into sum of simple ones

OR

$\Rightarrow$ we can split complicated input into sum of simple ones

(nothing more than "linearity")

4. Convolution with impulses

$$x(t) * \delta(t - \tau) = x(t - \tau)$$